

1. A system for performing a fast Fourier transform on N ordered inputs in n stages comprising:

a non-final stage calculating means for repetitively performing in-place butterfly calculations for n-1 stages;

5 a final stage calculating means for performing a final stage of butterfly calculations including:

a first loop means for performing a portion of the final stage butterfly calculations, the first loop means iterating on a table of first loop index values consisting of values that bit-reverse into themselves, the first loop means including control logic to  
10 select inputs for a set of butterfly calculations based on the first loop index values, performing the set of butterfly calculations, and storing butterfly calculation outputs in shuffled order in place of the selected inputs to result in a correct ordering of transform outputs; and

a second loop means for performing a remaining portion of the final stage  
15 butterfly calculations, the second loop means iterating on a table of second loop index value pairs consisting of two values that bit-reverse into each other, the second loop means including control logic to select inputs for two sets of butterfly calculations based on the two second loop index pair values respectively, performing two sets of butterfly calculations, and storing butterfly calculation outputs from a first one of the two sets of  
20 butterfly calculations in shuffled order in place of the inputs selected for a second one of the two sets of butterfly calculations and storing butterfly calculation outputs from the second one of the two sets of butterfly calculations in shuffled order in place of the inputs selected for the first one of the two sets of butterfly calculations to result in a correct ordering of transform outputs.

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2. The system of claim 1, wherein the final stage calculating means performs all butterfly calculations as radix-4 butterflies having four inputs and four outputs.

3. The system of claim 2, wherein N is a power of two.

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4. The system of claim 3, wherein the non-final stage calculating means performs a first stage of radix-8 butterfly calculations followed by  $n-2$  stages of radix-4 butterfly calculations.

5 5. The system of claim 3, wherein the first loop means iterates through a list of first loop index values between 0 and  $N/16-1$  that bit reverse into themselves.

6. The system of claim 5, wherein the first loop means includes control logic for selecting four groups of four consecutive inputs for each first loop iteration, the inputs  
10 being selected by transforming the first loop index value into four input indices by multiplying the first loop index value by four and successively adding  $N/4$  to result in four input indices, each group of four consecutive inputs being selected beginning with one input index, the four groups of four consecutive inputs being representable as a  $4 \times 4$  matrix.

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7. The system of claim 6, wherein the first loop means further includes control logic for performing four radix-4 butterfly calculations in each first loop iteration, one butterfly calculation being performed on each group of four consecutive inputs, the four radix-4 butterfly calculations generating four groups of four outputs, the outputs being  
20 representable as a  $4 \times 4$  matrix.

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8. The system of claim 7, wherein the first loop means further includes control logic to store the outputs in place of the inputs in shuffled order, the shuffled order resulting from a  $4 \times 4$  matrix transposition and subsequent swapping of two inner columns.

9. The system of claim 8, wherein control logic of the first loop means shuffles the order of the four groups of four inputs to the radix-4 butterfly calculations so as to generate outputs in shuffled order.

10. The system of claim 8, wherein control logic of the first loop means shuffles the order of the four groups of four outputs from the radix-4 butterfly calculations before storing.

5 11. The system of claim 3, wherein the second loop means iterates through a list of second loop index value pairs between 0 and  $N/16-1$  that bit reverse into each other.

12. The system of claim 11, wherein the second loop means includes control logic for selecting two sets of four groups of four consecutive inputs for each second loop iteration, the first set of inputs being selected by transforming a first value of a second loop index value pair into four input indices by multiplying the first value of the second loop index value pair by four and successively adding  $N/4$  to result in four input indices, each group of four consecutive inputs in the first set of inputs being selected beginning with one input index, the four groups of four consecutive inputs in the first set being representable as a  $4 \times 4$  matrix, the second set of inputs being selected by transforming a second value of the second loop index value pair into four input indices by multiplying the second value of the second loop index value pair by four and successively adding  $N/4$  to result in four input indices, each group of four consecutive inputs in the second set of inputs being selected beginning with one input index, the four groups of four consecutive inputs in the second set being representable as a  $4 \times 4$  matrix.

13. The system of claim 12, wherein the second loop means further includes control logic for performing two sets of four radix-4 butterfly calculations in each second loop iteration, one butterfly calculation being performed on each group of four consecutive inputs, the four radix-4 butterfly calculations generating two sets of four groups of four outputs, the outputs being representable as two  $4 \times 4$  matrices.

14. The system of claim 13, wherein the second loop means further includes control logic to store the outputs of each set of four radix-4 butterfly calculations in place of the inputs to the other set of four radix-4 butterfly calculations in shuffled order, the shuffled

order resulting from a 4 X 4 matrix transposition and subsequent swapping of two inner columns.

15. The system of claim 14, further comprising a twiddle factor storage element  
5 storing twiddle factors for application in the butterfly calculations, the twiddle factor storage element storing twiddle factors in groups of four, each group having an index and the groups being stored in bit reversed order based on the index.

16. The system of claim 2, wherein the non-final and final stage calculating means  
10 include a four-fold SIMD processor for performing four radix-4 butterfly calculations at a time.

17. In a system for calculating a fast Fourier transform on N ordered input elements  
having a plurality of calculation stages wherein N is a power of two, a method for  
15 performing a final calculation stage comprising:

performing a first iteration loop, the first iteration loop accepting input elements  
located using index values that bit-reverse into themselves, performing butterfly  
calculations on the inputs, and storing output elements in place of the input elements in a  
shuffled order, the shuffled order resulting in the output elements being ordered in the  
20 same manner as the input elements to the fast Fourier transform, the iteration loop  
iterating through all bit-reversal index values,

performing a second iteration loop, the second iteration loop accepting input  
elements located using pairs of index values that bit-reverse into each other, performing a  
first set of butterfly calculations using input elements located using a first index value of  
25 a bit-reversal pair, performing a second set of butterfly calculations using input elements  
located using a second index value of a bit-reversal pair, storing output elements from the  
first set of butterfly calculations in place of the input elements to the second set of  
butterfly calculations in shuffled order and storing output elements from the second set of  
butterfly calculations in place of the input elements to the first set of butterfly  
30 calculations in shuffled order, the order of the storing resulting in the output elements

being ordered in the same manner as the input elements to the fast Fourier transform, the iteration loop iterating through all bit-reversal pair index values.

18. The method of claim 17, wherein the final stage butterfly calculations are  
5 performed radix-4 butterflies having four inputs and four outputs.

19. The method of claim 18, wherein performing the first iteration loop includes  
iterating through a list of first loop index values between 0 and  $N/16-1$  that bit reverse  
into themselves.  
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20. The method of claim 19, wherein performing the first iteration loop includes  
selecting four groups of four consecutive inputs for each first loop iteration, the inputs  
being selected by transforming the first loop index value into four input indices by  
multiplying the first loop index value by four and successively adding  $N/4$  to result in  
15 four input indices, each group of four consecutive inputs being selected beginning with  
one input index, the four groups of four consecutive inputs being representable as a  $4 \times 4$   
matrix.

21. The method of claim 20, wherein performing the first iteration loop includes  
20 performing four radix-4 butterfly calculations in each first loop iteration, one butterfly  
calculation being performed on each group of four consecutive inputs, the four radix-4  
butterfly calculations generating four groups of four outputs, the outputs being  
representable as a  $4 \times 4$  matrix.

22. The method of claim 21, wherein performing the first iteration loop includes  
25 storing the outputs in place of the inputs in shuffled order, the shuffled order resulting  
from a  $4 \times 4$  matrix transposition and subsequent swapping of two inner columns.

23. The method of claim 22, wherein performing the first iteration loop includes  
30 shuffling the order of the four groups of four inputs to the radix-4 butterfly calculations  
so as to generate outputs in shuffled order.

24. The method of claim 23, wherein performing the first iteration loop includes shuffling the order of the four groups of four outputs from the radix-4 butterfly calculations before storing.

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25. The method of claim 18, wherein performing the second iteration loop includes iterating through a list of second loop index value pairs between 0 and  $N/16-1$  that bit reverse into each other.

10 26. The method of claim 25, wherein performing the second iteration loop includes selecting two sets of four groups of four consecutive inputs for each second loop iteration, the first set of inputs being selected by transforming a first value of a second loop index value pair into four input indices by multiplying the first value of the second loop index value pair by four and successively adding  $N/4$  to result in four input indices,  
15 each group of four consecutive inputs in the first set of inputs being selected beginning with one input index, the four groups of four consecutive inputs in the first set being representable as a  $4 \times 4$  matrix, the second set of inputs being selected by transforming a second value of the second loop index value pair into four input indices by multiplying the second value of the second loop index value pair by four and successively adding  $N/4$   
20 to result in four input indices, each group of four consecutive inputs in the second set of inputs being selected beginning with one input index, the four groups of four consecutive inputs in the second set being representable as a  $4 \times 4$  matrix.

27. The method of claim 26, performing the second iteration loop includes  
25 performing two sets of four radix-4 butterfly calculations in each second loop iteration, one butterfly calculation being performed on each group of four consecutive inputs, the four radix-4 butterfly calculations generating two sets of four groups of four outputs, the outputs being representable as two  $4 \times 4$  matrices.

30 28. The method of claim 27, performing the second iteration loop includes storing the outputs of each set of four radix-4 butterfly calculations in place of the inputs to the other

set of four radix-4 butterfly calculations in shuffled order, the shuffled order resulting from a 4 X 4 matrix transposition and subsequent swapping of two inner columns.

29. The method of claim 28, further comprising storing twiddle factors for application in the butterfly calculations, the twiddle factor storage element storing twiddle factors in groups of four, each group having an index and the groups being stored in bit reversed order based on the index.

30. The method of claim 18, wherein performing radix-4 butterfly calculations includes performing four radix-4 butterfly calculations at a time using a four-fold SIMD processor.

31. A computer program product for performing a final calculation stage in a system for calculating a fast Fourier transform on N ordered input elements having a plurality of calculation stages wherein N is a power of two, comprising:

a computer readable medium containing computer readable code for performing a first iteration loop, the first iteration loop accepting input elements located using index values that bit-reverse into themselves, performing butterfly calculations on the inputs, and storing output elements in place of the input elements in a shuffled order, the shuffled order resulting in the output elements being ordered in the same manner as the input elements to the fast Fourier transform, the iteration loop iterating through all bit-reversal index values,

a computer readable medium containing computer readable code performing a second iteration loop, the second iteration loop accepting input elements located using pairs of index values that bit-reverse into each other, performing a first set of butterfly calculations using input elements located using a first index value of a bit-reversal pair, performing a second set of butterfly calculations using input elements located using a second index value of a bit-reversal pair, storing output elements from the first set of butterfly calculations in place of the input elements to the second set of butterfly calculations in shuffled order and storing output elements from the second set of butterfly calculations in place of the input elements to the first set of butterfly calculations in

shuffled order, the order of the storing resulting in the output elements being ordered in the same manner as the input elements to the fast Fourier transform, the iteration loop iterating through all bit-reversal pair index values.

5     32.     The computer program product of claim 31, wherein the final stage butterfly calculations are performed radix-4 butterflies having four inputs and four outputs.

33.     The computer program product of claim 32, wherein performing the first iteration loop includes iterating through a list of first loop index values between 0 and  $N/16-1$  that  
10     bit reverse into themselves.

34.     The computer program product of claim 33, wherein performing the first iteration loop includes selecting four groups of four consecutive inputs for each first loop iteration, the inputs being selected by transforming the first loop index value into four input  
15     indices by multiplying the first loop index value by four and successively adding  $N/4$  to result in four input indices, each group of four consecutive inputs being selected beginning with one input index, the four groups of four consecutive inputs being representable as a  $4 \times 4$  matrix.

20     35.     The computer program product of claim 34, wherein performing the first iteration loop includes performing four radix-4 butterfly calculations in each first loop iteration, one butterfly calculation being performed on each group of four consecutive inputs, the four radix-4 butterfly calculations generating four groups of four outputs, the outputs being representable as a  $4 \times 4$  matrix.

25     36.     The computer program product of claim 35, wherein performing the first iteration loop includes storing the outputs in place of the inputs in shuffled order, the shuffled order resulting from a  $4 \times 4$  matrix transposition and subsequent swapping of two inner columns.

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37. The computer program product of claim 36, wherein performing the first iteration loop includes shuffling the order of the four groups of four inputs to the radix-4 butterfly calculations so as to generate outputs in shuffled order.

5 38. The computer program product of claim 37, wherein performing the first iteration loop includes shuffling the order of the four groups of four outputs from the radix-4 butterfly calculations before storing.

39. The computer program product of claim 38, wherein performing the second  
10 iteration loop includes iterating through a list of second loop index value pairs between 0 and  $N/16-1$  that bit reverse into each other.

40. The computer program product of claim 39, wherein performing the second iteration loop includes selecting two sets of four groups of four consecutive inputs for  
15 each second loop iteration, the first set of inputs being selected by transforming a first value of a second loop index value pair into four input indices by multiplying the first value of the second loop index value pair by four and successively adding  $N/4$  to result in four input indices, each group of four consecutive inputs in the first set of inputs being selected beginning with one input index, the four groups of four consecutive inputs in the  
20 first set being representable as a  $4 \times 4$  matrix, the second set of inputs being selected by transforming a second value of the second loop index value pair into four input indices by multiplying the second value of the second loop index value pair by four and successively adding  $N/4$  to result in four input indices, each group of four consecutive inputs in the second set of inputs being selected beginning with one input index, the four groups of  
25 four consecutive inputs in the second set being representable as a  $4 \times 4$  matrix.

41. The computer program product of claim 40, performing the second iteration loop includes performing two sets of four radix-4 butterfly calculations in each second loop iteration, one butterfly calculation being performed on each group of four consecutive  
30 inputs, the four radix-4 butterfly calculations generating two sets of four groups of four outputs, the outputs being representable as two  $4 \times 4$  matrices.

42. The computer program product of claim 41, performing the second iteration loop includes storing the outputs of each set of four radix-4 butterfly calculations in place of the inputs to the other set of four radix-4 butterfly calculations in shuffled order, the  
5 shuffled order resulting from a 4 X 4 matrix transposition and subsequent swapping of two inner columns.

43. The computer program product of claim 42, further comprising storing twiddle factors for application in the butterfly calculations, the twiddle factor storage element  
10 storing twiddle factors in groups of four, each group having an index and the groups being stored in bit reversed order based on the index.

44. The computer program product of claim 32, wherein performing radix-4 butterfly calculations includes performing four radix-4 butterfly calculations at a time using a four-  
15 fold SIMD processor.

45. A computer system for system for performing a fast Fourier transform on N ordered inputs in n stages comprising:  
a four-fold SIMD processor;  
20 a non-final stage calculating means for repetitively performing in-place butterfly calculations for n-1 stages, the butterfly calculations being performed in groups of four simultaneous butterflies on the four-fold SIMD processor;  
a final stage calculating means for performing a final stage of butterfly calculations on the four-fold SIMD processor including:  
25 a first loop means for performing a portion of the final stage butterfly calculations, the first loop means iterating on a table of first loop index values consisting of values that bit-reverse into themselves, the first loop means including control logic to select inputs for four simultaneous radix-4 butterfly calculations based on the first loop index values, performing the four simultaneous radix-4 butterfly calculations, and storing  
30 butterfly calculation outputs in shuffled order in place of the selected inputs to result in a correct ordering of transform outputs; and

a second loop means for performing a remaining portion of the final stage butterfly calculations, the second loop means iterating on a table of second loop index value pairs consisting of two values that bit-reverse into each other, the second loop means including control logic to select inputs for two sets of four simultaneous radix-4 butterfly calculations based on the two second loop index pair values respectively, performing the two sets of four simultaneous radix-4 butterfly calculations, and storing butterfly calculation outputs from a first one of the two sets of butterfly calculations in shuffled order in place of the inputs selected for a second one of the two sets of butterfly calculations and storing butterfly calculation outputs from the second one of the two groups of butterfly calculations in shuffled order in place of the inputs selected for the first one of the two groups of butterfly calculations to result in a correct ordering of transform outputs.

46. A method for calculating a fast Fourier transform comprising:
- accessing an ordered set of N transform inputs;
  - performing one or more non-final stages of butterfly calculations, each of the one or more stages accepting N inputs and transforming them into stage outputs and storing the outputs in place of the outputs in a bit-reversed order;
  - performing a final stage of butterfly calculations using the outputs of a previous stage of butterfly calculations as inputs to the final stage of butterfly calculations, the final stage of butterfly calculations including:
    - (a) a first loop iterated through a list of first loop index values between 0 and N/16-1 that bit reverse into themselves, the first loop including:
      - (i) selecting four groups of four consecutive inputs, the inputs being selected by transforming the first loop index value into four input indices by multiplying the first loop index value by four and successively adding N/4 to result in four input indices, each group of four consecutive inputs being selected beginning with one input index, the four groups of four consecutive inputs being representable as a 4 X 4 matrix;
      - (ii) performing four radix-4 butterfly calculations, one calculation for each group of four inputs; and

- (iii) storing the outputs in place of the inputs in shuffled order, the shuffled order resulting from a 4 X 4 matrix transposition and subsequent swapping of two inner columns; and
- (b) a second loop iterated through a list of second loop index pair values, the  
5 second loop index pair values being each pair of values between 0 and N/16-1 that bit reverse into each other, the including loop including:
  - (i) selecting two sets of four groups of four consecutive inputs, the  
inputs being selected by transforming each value in the second loop index pair  
into four input indices by multiplying each second loop index pair value by four  
10 and successively adding N/4 to result in two sets of four input indices, each group  
of four consecutive inputs being selected beginning with one input index, the two  
sets of four groups of four consecutive inputs being representable as two 4 X 4  
matrices;
  - (ii) performing two sets of four radix-4 butterfly calculations, one  
15 calculation for each group of four inputs; and
  - (iii) storing the outputs of each set of four radix-4 butterfly calculations  
in place of the inputs to the other set of four radix-4 butterfly calculations in  
shuffled order, the shuffled order resulting from a 4 X 4 matrix transposition and  
subsequent swapping of two inner columns;
- 20 wherein following the second loop, outputs from the fast Fourier transform are  
correctly ordered.